

Problem Set 3

Due at the start of class Wednesday, Feb. 20th

1. Consider the production function $Q=K^aL^b$.
 - a. What restrictions must you place on a and b for the function to exhibit diminishing marginal returns to labor, holding capital constant?
 - b. Show that if $a + b < 1$, the function exhibits decreasing returns to scale. What does decreasing returns to scale imply about costs?

2. The production function for "Itchy and Scratchy" cartoons is $Q=K*L$, where Q = minutes of cartoon output per week, K = capital, and L = labor.
 - a. Draw the isoquants for $Q=1$, $Q=4$, and $Q=16$.
 - b. If wages are 1 and the rate of capital rental is 1, what is the minimum-cost method (combination of K and L) of producing 4 minutes of cartoon per week? What is the total cost of this level of production?
 - c. If the firm wants to produce 16 minutes per week but cannot expand capital beyond two units, what are total costs?
 - d. If the firm can alter K and L , how does it choose to produce 16 minutes of cartoon?
 - e. Draw the firm's short run ($K=2$) and long run (K variable) total cost curves.

3. The production function for widgets is $Q=F(K,L)=K^aL^b$. The cost of producing widgets is $rK + wL$, where r = the rental rate on capital and w = the wage paid to labor. Derive the input demand functions $L(w,r,Q)$ and $K(w,r,Q)$.
 - a. If $r=1$, $w=2$, $a = (1/3)$, and $b = (2/3)$, what are the cost-minimizing combinations of K and L for producing 5, 10, 15, and 20 units of output? What are the costs of producing these levels of output? How would you characterize returns to scale for this firm?
 - b. Do exercise 8.a. again with $a=(2/3)$ and $b=(2/3)$.

4. A firm produces 100 units of ice cream using capital and labor.
 - a. If the price of labor rises and the price of capital remains constant, what happens to the cost of production?
 - b. If the price of labor rises and the price of capital falls, what happens to production cost? Consider various production technologies and explain your assumptions.

5. A firm in a competitive industry has the following cost curves: Short-run total cost = $200 + 60q - 10q^2 + (2/3)q^3$, and the market price of output, $P = 38$.
- What is the short run equilibrium q for the firm? Use calculus and draw a diagram.
 - What are the firm's profits in the short-run equilibrium? Show them on the diagram. Is the firm making or losing money?
 - Should the firm shut down in the short run at a price of 38?
 - If the price of output were 68, how would answers to a. and b. change?
6. Suppose that supply and demand in the corn market are described by the following curves: Demand: $P = 100 - 2Q$ Supply: $P = 25 + 3Q$
- Find the equilibrium price and quantity without any price support. What are consumer surplus, producer surplus, and deadweight loss?
 - The government announces that the price of corn will be supported at 85. In order to accomplish this, the government stands ready to buy enough corn at a price of 85 so that at the intersection of supply and total demand, the equilibrium price is 85. At the support price, what quantity do private consumers buy? What quantity does the government buy? What does the program cost the government?
 - What effect does the program have on producer surplus, consumer surplus, and deadweight loss?
 - How does the gain to farmers compare with the cost of the program? Do you think this is a good program?
7. A firm in a competitive market produces widgets with the production function $q = 2\sqrt{L}$. L is hours of labor, the price of widgets is \$10, and the prevailing wage is \$5 per hour.
- Find the firm's supply function. Represent it on a graph with the firm's demand function.
 - What is the firm's profit-maximizing output and labor input?
8. Briefly analyze the following statement: "California recently raised its cigarette tax by 25 cents per pack. We would therefore expect the price of cigarettes to rise by 25 cents per pack." Diagrams will be helpful. Justify your assumptions.