Production Theory

- Looks like consumer theory
- Big Questions for firms
  - How much to produce?
  - With what combination of inputs?
- Basic Tool: production function
  - Represents how firms turn **inputs** into **output**
  - E.g. q=F(K,L)
    - "output depends on the amount of, say, labor and capital"
  - As a characterization of firms, it neglects some issues
    - "where's my boss, the idiot?"

Production Function

- Shows the ***maximum*** amount of output that can be produced with each combination of K and L.
- Production isoquant
  - Locus of (K, L) combinations that produce a constant amount of output
  - {(K,L)| F(K,L)=c}

Production Isoquants

- In "input space"

- Isoquants are similar to indifference curves, <u>except</u> that each isoquant is associated with a particular level of output.

Short Run and Long Run

- In the LR, all inputs can be varied.
- In the SR, only some inputs can be varied
- Usual assumption
  - With (K,L), labor is variable in the SR
  - When does this assumption make sense?

2 Tools, 3 Questions

- Tools
  - Isoquant graphs in input space
  - The production function q=F(K,L)
- Questions
  - How does marginal product vary inputs?
    - one input at a time; usually "diminishing marginal product"
  - How substitutable are inputs?
  - How does output vary with the scale of production
    - Change both inputs by, say, x. How much does output change?
    - "returns to scale"

1: Marginal Product

- Production with one variable input
  - Vary labor, hold capital constant
    - “Usual” shape
    - Totals above, marginal, averages below

  - Marginal product=Slope of total product
    - =∂F/∂L
  - Average product=slope of ray from origin to a point on the TP curve
    - =F(K,L)/L

Example

- Consider $q=F(K,L)=K^aL^b$,
  - how does q vary with L, holding K constant?
  - $\partial q/\partial L = bK^aL^{b-1} > 0$
    - More labor generates more output
  - $\partial^2 q/\partial L^2 = b(b-1)K^aL^{b-2} < 0$ if $0 < b < 1$
    - The growth in output with increased labor declines as more labor is added
    - ***Diminishing returns to labor*** if $0 < b < 1$
  - Again, holding K fixed corresponds to the short run

Illustrating MP in Input Space

- Hold one input constant, vary the other.
    - Move across isoquants: {K=3; L=1, L=2, L=3}

    - How does q vary with L?

2: Substitution among Inputs

- Slope of the isoquant indicates the quantity of one input that can be traded for another input, while keeping output constant
- Marginal rate of technical substitution (MRTS)= -(slope of isoquant)
- E.g. q=F(K,L)=c along an isoquant
- Totally differentiating yields:
  - $0=(\partial F/\partial K)dK + (\partial F/\partial L)dL$
  - $MRTS=MP_L/MP_K=-(dK/dL)$=-(slope of the isoquant)

- Shape of isoquant reflects that the more capital there is, the more productive labor is
- E.g. $q=K^aL^b$

Substitutability

- Graphs convey intuition

- The flatter the isoquant, the more that substitution is possible

3: Returns to Scale

- If we double ***both*** inputs, what happens to output?
- Does it …double?
  - Constant returns to scale (CRS)
- …more than double?
  - Increasing returns to scale (IRS)
- …Less than double?
  - Decreasing returns to scale (DRS)
- If q=F(K,L), is 2q > < F(2K,2L)?

Returns to Scale in Input Space

Returns to Scale Mathematically

- Cases
  - $tF(K,L)>F(tK,tL) \Leftrightarrow$ DRS
  - $tF(K,L)>F(tK,tL) \Leftrightarrow$ IRS
  - $tF(K,L)=F(tK,tL) \Leftrightarrow$ CRS
- E.g. $q=K^aL^b$
  - When does it exhibit constant, increasing, decreasing returns?